

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS (SYLLABUS D)

4024/21
Paper 2
October/November 2010
2 hours 30 minutes
Additional Materials: Answer Booklet/Paper
Graph paper ( 1 sheet)
Electronic calculator Geometrical instruments

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

## Section A

Answer all questions.

## Section B

Answer any four questions.
Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100 .

## Section A [52 marks]

Answer all questions in this section.

1 (a) $P=\frac{7 Q}{4}+15$
(i) Find $P$ when $Q=-40$.
(ii) Express $Q$ in terms of $P$.
(b) Factorise completely
(i) $7 c^{2}-28 d^{2}$,
(ii) $3 x^{2}-7 x-6$.
(c) Solve the equation $\frac{4}{7-y}=5$.

2 (a)

$A B$ is vertical and $B C$ is horizontal.
$A B=49 \mathrm{~m}$ and $B C=180 \mathrm{~m}$.
(i) Calculate $B \hat{A} C$.
(ii) State the value of the angle of depression of $C$ from $A$.
(b) $S$ and $P$ are two positions at sea and $L$ is a Lighthouse.

$S$ is due West of $L$ and $P$ is due South of $L$.
$S L=1200 \mathrm{~m}$ and $S P=1300 \mathrm{~m}$.
(i) Calculate $L P$.
(ii) Calculate the bearing of $S$ from $P$.
(iii) A boat sailed from $S$ to $P$.

It left $S$ at 1556 and reached $P$ at 1604.
Calculate its speed in kilometres per hour.

3 (a)


The points $P, T, U$ and $V$ lie on Circle I, and the points $P, Q, R, S$ and $T$ lie on Circle II. $U P Q$ and $V P R$ are straight lines.
$V \hat{T} U=38^{\circ}$ and $T \hat{S} R=106^{\circ}$.
Find
(i) $V \hat{P} U$,
(ii) $Q \hat{T} R$,
(iii) $T \hat{P} R$,
(iv) UPT .
(b)

$A$ and $B$ are points on the circle, centre $O$.
$C A$ is a tangent at $A$ and $B O C$ is a straight line.
$A \hat{C} B=x^{\circ}$ and $B \hat{A} O=y^{\circ}$.
Find an expression for $y$ in terms of $x$.

4 (a) $\mathscr{E}=\{x: x$ is an integer and $10 \leqslant x \leqslant 21\}$
$E=\{x: x$ is an even number $\}$
$M=\{x: x$ is a multiple of 5$\}$
$P=\{x: x$ is a prime number $\}$


The Venn diagram shows the Universal set $\mathscr{E}$ and the subsets $E$ and $M$.
Two elements of $\mathscr{E}$ are shown in their appropriate subsets.
(i) Copy the Venn diagram shown above and draw the subset $P$.
(ii) Write the remaining 10 elements of $\mathscr{E}$ in the appropriate subsets of your Venn diagram.
(b) $\mathscr{E}=\{$ letters of the alphabet $\}$
$J=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}, \mathrm{g}\}$
$K=\{\mathrm{a}, \mathrm{e}, \mathrm{i}, \mathrm{o}, \mathrm{u}\}$
(i) Find $n(J \cup K)$.
(ii) Find $J \cap K^{\prime}$.
(iii) Given that $L \subset J$ and $L \subset K$, state the maximum value of $n(L)$.
(iv) A letter is picked at random from $K$.

Find the probability that it is not an element of $J$.
(c) In a school, 24 students are members of the Athletics Club, 30 students are members of the Basketball Club and 36 students are members of the Drama Club.
5 students are members of all three clubs.
12 students are members of both the Athletics Club and the Drama Club.
13 students are members of both the Basketball Club and the Athletics Club.
19 students are members of both the Drama Club and the Basketball Club.
(i) How many students are members of the Basketball Club, but are not members of the other two Clubs?
(ii) How many students are members of one or more of the Clubs?

5 Archie bought a car from a dealer.
The dealer sold the car for $\$ 7040$.
Archie paid a deposit of $\$ 1760$.
(a) Calculate the deposit as a percentage of the dealer's selling price.
(b) Archie took out a loan agreement to repay the outstanding balance of $\$ 5280$.

He paid $\$ 212.67$ per month for 36 months.

Calculate
(i) the amount of interest that he paid,
(ii) the rate of simple interest per year that had been charged.
(c) When the dealer sold the car, he made a profit of $30 \%$.

Calculate the profit, correct to the nearest dollar, that the dealer made on the sale of this car.

6 (a) The results of a survey of the number of televisions in 40 households are given in the table below.

| Number of televisions | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Number of households | 8 | 17 | 12 | 3 |

(i) Calculate the mean number of televisions per household.
(ii) The survey was extended to include 5 more households.

It was found that none of these had a television.
For the 45 households, calculate the mean number of televisions per household.
(b) 120 children were asked which colour they liked best. 59 said red, 21 said blue and the rest said yellow.
(i) Using a radius of 4 cm , draw an accurate pie chart to show these results.
(ii) Out of the children who liked blue best, $\frac{5}{7}$ were wearing blue socks.

How many of the children who liked blue best were not wearing blue socks?

## Section B [48 marks]

Answer four questions in this section.
Each question in this section carries 12 marks.


30

The diagram shows an open rectangular tank with base 20 cm by 30 cm . The tank contains $9600 \mathrm{~cm}^{3}$ of water.
(a) (i) State the number of litres of water in the tank.
(ii) Calculate the depth of the water.
(iii) Calculate the total surface area of the tank that is in contact with the water.
(iv) The water had entered the tank through a circular pipe of radius 0.8 cm .

It flowed through the pipe at 25 centimetres per second.
How long did the $9600 \mathrm{~cm}^{3}$ of water take to enter the tank?
Give your answer correct to the nearest second.
(b) [Volume of a sphere $=\frac{4}{3} \pi r^{3}$ ]

250 identical spheres are placed in the bottom of the tank.
Each sphere has a volume of $2.6 \mathrm{~cm}^{3}$.
(i) Calculate by how much the water level in the tank will rise.

Give your answer in millimetres.
(ii) Calculate the radius of one of these spheres.

## 8 Answer the WHOLE of this question on a sheet of graph paper.

The variables $x$ and $y$ are connected by the equation

$$
y=x^{2}-1 .
$$

(a) Copy and complete the following table of values.

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |  |  |  |  |  |

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal $x$-axis for $-4 \leqslant x \leqslant 4$. Using a scale of 1 cm to represent 1 unit, draw a vertical $y$-axis for $-2 \leqslant y \leqslant 16$.

On your axes, plot the points from your table and join them with a smooth curve.
(c) It is given that $\mathrm{f}(x)=\frac{x+7}{2}$.
(i) Using the same axes, draw the graph of $y=\mathrm{f}(x)$.
(ii) Find $\mathrm{f}^{-1}(3)$.
(iii) (a) Write down the $x$ coordinates of the two points where the graphs intersect.
(b) Find the equation that is satisfied by these two values of $x$.

Express your answer in the form $a x^{2}+b x+c=0$, where $a, b$ and $c$ are integers.

9 (a)

$A B C$ is a triangle in which $A \hat{B} C=34^{\circ}$ and $A C=15 \mathrm{~cm}$.
$B C D$ is a straight line and $A \hat{C} D=60^{\circ}$.
Find
(i) $B \hat{A} C$,
(ii) $B C$.
(b)


The shaded part of the diagram shows a lake. $P$ and $Q$ are points at each end of the lake. $R$ is a point on the same level ground as $P$ and $Q$.
$P R=55 \mathrm{~m}, Q R=70 \mathrm{~m}$ and $P \hat{R} Q=112^{\circ}$.
(i) Calculate $P Q$.
(ii) A plan is drawn using a scale of 1 centimetre to 5 metres.
(a) State the lengths of $P R$ and $Q R$ on the plan.
(b) Find the area of triangle $P Q R$ on the plan.
(c) The area of the lake on the plan is $32.4 \mathrm{~cm}^{2}$.

Calculate the actual area, in square metres, of the lake.

10 (a)


In the diagram, $A B C D$ is a quadrilateral where $\overrightarrow{A B}=\binom{4}{8}, \overrightarrow{B C}=\binom{2}{0}$ and $\overrightarrow{C D}=\binom{8}{-12}$.
$E, F, G$ and $H$ are the midpoints of $A B, B C, C D$ and $D A$ respectively.
(i) Find $\overrightarrow{A D}$.
(ii) Calculate $|\overrightarrow{A D}|$.
(iii) Show that $E F$ and $H G$ are opposite sides of a parallelogram.
(b) The diagram shows triangle $P$.


Triangle $Q$ has vertices $(-2,4),(6,0)$ and $(6,4)$.
Describe fully the single transformation that maps triangle $P$ onto triangle $Q$.
(c) The transformation represented by the matrix $\left(\begin{array}{ll}5 & 2 \\ 0 & 3\end{array}\right)$ maps the square $O(0,0), U(1,0), V(1,1)$,
$W(0,1)$ onto $O U^{\prime} V^{\prime} W^{\prime}$.
(i) Find the coordinates of $U^{\prime}, V^{\prime}$ and $W^{\prime}$.
(ii) Find the matrix that represents the transformation that maps $O U^{\prime} V^{\prime} W^{\prime}$ onto $O U V W$.

11 A gardener uses No-Weed to kill weeds in his lawn.
(a) He mixes 15 ml of No-Weed with 5 litres of water.

Calculate the ratio of No-Weed to water in the mixture.
Express the ratio in the form $m: n$, where $m$ and $n$ are the smallest possible whole numbers. [1]
(b) The gardener's lawn is a rectangle of length 27 metres and width 25 metres.
(i) 15 ml of No -Weed are required to kill weeds in $10 \mathrm{~m}^{2}$ of lawn. No-Weed is sold in 500 ml bottles.
(a) How many bottles does the gardener need to buy?
(b) How many millilitres are not used?
(ii) The lawn is to be made larger.

The length is increased by $3 x$ metres and the width by $x$ metres.
The area of the larger lawn is twice that of the original lawn.
(a) Form an equation in $x$ and show that it reduces to $x^{2}+34 x-225=0$.
(b) Solve the equation $x^{2}+34 x-225=0$, giving each answer correct to 2 decimal places.
(c) Hence find the length of the larger lawn.

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